Eur. Phys. J. C 44, 287–291 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02356-0

THE EUROPEAN
PHYSICAL JOURNAL C

Vector bosons in the expanding universe

Y. Sucu[a], N. Ünal[b]

Physics Department, Akdeniz University, P. K. 510 Antalya, 07058 Turkey

Received: 20 May 2005 /
Published online: 30 August 2005 –

Abstract. We exactly solve the relativistic wave equation for vector bosons in the expanding universe and show that the current of the vector bosons in this background is rapidly oscillating in early time. Additionally, we derive the solutions of the Proca equation from the solutions of the Duffin–Kemmer–Petiau (DKP) equations in the same background and obtain the massless-particle, photon, solutions by taking the $m^2 \to 0$ limit of these solutions.

PACS. 03.65.Pm, 04.60.-m, 98.80.Cq

1 Introduction

Relativistic wave equations are useful tools for investigating the physical behaviors of particles in a curved spacetime. In particular, it is important to consider these equations in the expanding universe, which has considerable importance in astrophysics and cosmology. These equations, especially the Dirac equation, have been extensively studied in curved spacetime for seventy years [1–6]. On the other hand, the Duffin–Kemmer–Petiau (DKP) equation, which is a first-order equation and represents vector bosons [7–9], is almost as old as the Dirac equation, but has only recently begun to be studied in a curved spacetime [10, 11], as a family of DKP oscillator [12, 13] and some problems in nuclear physics [14, 15]. In all of these studies, the DKP equation has two parts: spin-0 and spin-1 particles.

Only the spin-1 sector of the DKP equation is obtained from the quantization of a classical model of the *zitterbewegung*, as well as the Dirac equation and the other higher-spin wave equations [16]. The quantization procedure requires that the wavefunction is a symmetric spinor of rank 2. The wavefunction does not then have a spin-zero sector [17], and the wave function in this case is also represented as the direct product of the two Dirac spinors.

A study of pair creation in Robertson–Walker (RW) metric was first investigated by Parker [18], and later by Barut and Duru [19]. In our previous study, we solved the massless spin-1, photon, wave equation in the RW metric and showed that, as a result of conformal invariance, there is no pair creation in this case [20]. However, the conformal invariance is broken for the massive case. Therefore, in the evaluation of the currents it becomes important to understand the pair-creation process for the massive vector bosons in this background.

In this study, we are interested in the behavior of the spin-1 massive particles, $W^{\pm}$ bosons, in the expanding universe. So, we find an exact solution to the DKP equation in the spatially flat RW metric and compute the currents of the vector bosons from the solutions. We show that the currents are rapidly oscillating in early times and that the vacuum is not stable. We also discuss the massless particle limit of these equations and their solutions, first by deriving the solutions of the Proca equation from the solutions of the DKP equation and secondly by evaluating the limit $m^2 \to 0$.

In Sect. 2, we derive the spin-1 sector of the DKP equation in spatially flat RW spacetime. In Sect. 3, we find the exact solutions of the DKP equation in terms of Bessel functions, and in Sect. 4, we derive the expression of the current for the spin-1 particles and discuss the time dependence of the current. In the Conclusion, we write the Proca equation from the DKP equation in spatially flat RW spacetime and derive the massless-particle limit of these equations, the DKP and the Proca equations, and their solutions.

2 Spin-1 wave equation in RW spacetime

We start by writing the relativistic wave equation for the spin-1 particle in a curved spacetime, which is obtained from the quantization of the classical model of the zitterbewegung [16, 17]. It is

$$\left\{ i \left[\gamma^{\mu}(x) \otimes 1 + 1 \otimes \gamma^{\mu}(x) \right] \left[\partial_{\mu} - \Gamma_{\mu}^{\mathrm{DKP}}(x) \right] - 2m \right\}_{\alpha\beta,\gamma\delta} \times \Psi_{\gamma\delta}(x) = 0 \,, \tag{1}$$

where $\frac{1}{2}(\gamma^{\mu} \otimes 1 + 1 \otimes \gamma^{\mu})$ are called Kemmer matrices, β^{μ}, and $\Gamma_{\mu}^{\mathrm{D}KP}$ is the spin connection for the DKP spin-1

[a] e-mail: ysucu@akdeniz.edu.tr
[b] e-mail: nuriunal@akdeniz.edu.tr

particle wave equation and is given by

$$\Gamma_{\mu}^{\mathrm{DKP}} = \Gamma_{\mu}(x) \otimes 1 + 1 \otimes \Gamma_{\mu}(x) \,. \tag{2}$$

In (2), $\Gamma_{\mu}(x)$ is the spin connection for the spin-$\frac{1}{2}$ particles and is given in terms of the spacetime-dependent Dirac matrices, $\gamma^{\mu}(x)$, by

$$\Gamma_{\mu}(x) = -\frac{1}{8}\left[\gamma^{\nu}(x), \gamma_{\nu}(x)_{;\mu}\right] , \tag{3}$$

where the covariant derivative is denoted by the semicolon.

In this model, the wavefunction $\psi_{\alpha\beta}$ is the symmetric spinor of rank 2 and is represented as the direct product of two Dirac wavefunctions and the quantization of the classical system requires that $\psi_{\alpha\beta}$ should be symmetric with respect to the indices $\alpha\beta$, where the first (second) indices correspond to the first (second) set of the Dirac matrices. For thisreason, $\psi_{\alpha\beta}$ has 10 components and (1) corresponds to the spin-1 sector of the DKP equation.

The spatially flat RW metric is

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 \,, \tag{4}$$

where $a^2(t)$ is an expansion parameter that depends on time. Then, the spacetime-dependent Dirac matrices in terms of the constant Dirac matrices are

$$\gamma^0(x) = \gamma_0 \,, \quad \gamma^i(x) = -\frac{1}{a}\gamma_i \,. \tag{5}$$

Using (3) and (5), the spin connections for the spin $\frac{1}{2}$ particle are

$$\Gamma_0 = 0 \,, \tag{6}$$

$$\Gamma_i = \frac{1}{2}\dot{a}\gamma_0\gamma_i \,. \tag{7}$$

The wave equation for the spin-1 particle in spatially flat RW metric is

$$\left[H - 2m \cdot 1 \otimes 1\right] \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{bmatrix} = 0 \,, \tag{8}$$

where Ψ_1, Ψ_2, Ψ_3, and Ψ_4 are the four component spinors and the Hamiltonian H is

$$H = \tag{9}$$

$$\begin{bmatrix} D & -1 \otimes \vec{\sigma} \cdot \frac{i\vec{\nabla}}{a} & -\vec{\sigma} \otimes 1 \cdot \frac{i\vec{\nabla}}{a} & -i\frac{\dot{a}}{a}\sigma_i \otimes \sigma_i \\ 1 \otimes \vec{\sigma} \cdot \frac{i\vec{\nabla}}{a} & 0 & 0 & -\vec{\sigma} \otimes 1 \cdot \frac{i\vec{\nabla}}{a} \\ \vec{\sigma} \otimes 1 \cdot \frac{i\vec{\nabla}}{a} & 0 & 0 & -1 \otimes \vec{\sigma} \cdot \frac{i\vec{\nabla}}{a} \\ i\frac{\dot{a}}{a}\sigma_i \otimes \sigma_i & \vec{\sigma} \otimes 1 \cdot \frac{i\vec{\nabla}}{a} & 1 \otimes \vec{\sigma} \cdot \frac{i\vec{\nabla}}{a} & -D \end{bmatrix} ,$$

where D is

$$D = 2i \cdot 1 \otimes 1 \left[\left(\partial_t + \frac{3\dot{a}}{2a}\right)\right] . \tag{10}$$

In (9), the Hamiltonian is written as the direct product of the 2×2 Pauli matrices. In a similar way, the spinors are also denoted by two indices, and because of the quantization condition we denote the total 10 components of the Ψ_1, Ψ_2, Ψ_3, and Ψ_4 as

$$\Psi_1(\vec{x}, t) = e^{i\vec{k}\cdot\vec{x}} \begin{bmatrix} \Phi_{1+}(t) \\ \Phi_{10}(t) \\ \Phi_{10}(t) \\ \Phi_{1-}(t) \end{bmatrix} ,$$

$$\Psi_3(\vec{x}, t) = e^{i\vec{k}\cdot\vec{x}} \begin{bmatrix} \Phi_{2+}(t) \\ \Phi_{2\tilde{0}}(t) \\ \Phi_{20}(t) \\ \Phi_{2-}(t) \end{bmatrix} ,$$

$$\Psi_2(\vec{x}, t) = e^{i\vec{k}\cdot\vec{x}} \begin{bmatrix} \Phi_{2+}(t) \\ \Phi_{20}(t) \\ \Phi_{2\tilde{0}}(t) \\ \Phi_{2-}(t) \end{bmatrix} ,$$

$$\Psi_4(\vec{x}, t) = e^{i\vec{k}\cdot\vec{x}} \begin{bmatrix} \Phi_{4+}(t) \\ \Phi_{40}(t) \\ \Phi_{40}(t) \\ \Phi_{4-}(t) \end{bmatrix} ,$$

where we can separate the momentum eigenfunctions, $\exp\left(i\vec{k}\cdot\vec{x}\right)$ since $a(t)$ is a function of t only.

We first obtain the following first-order equations by adding and subtracting these equations for the momentum eigenstates. We then find that, by choosing the momentum in the z-direction only, $\vec{k} = (0, 0, k)$, for simplicity, the explicit forms of the 10 equations are

$$i\left(\partial_t + 2\frac{\dot{a}}{a}\right) \begin{bmatrix} (\Phi_1 - \Phi_4)_+ \\ (\Phi_1 - \Phi_4)_0 \\ (\Phi_1 - \Phi_4)_- \end{bmatrix} + \frac{k}{a} \begin{bmatrix} 0 \\ (\Phi_{20} - \Phi_{2\tilde{0}}) \\ 0 \end{bmatrix} = m \begin{bmatrix} (\Phi_1 + \Phi_4)_+ \\ (\Phi_1 + \Phi_4)_0 \\ (\Phi_1 + \Phi_4)_- \end{bmatrix} , \tag{11}$$

$$i\left(\partial_t + \frac{\dot{a}}{a}\right) \begin{bmatrix} (\Phi_1 + \Phi_4)_+ \\ (\Phi_1 + \Phi_4)_0 \\ (\Phi_1 + \Phi_4)_- \end{bmatrix} - \frac{k}{a} \begin{bmatrix} 2\Phi_{2+} \\ 0 \cdot (\Phi_{20} + \Phi_{2\tilde{0}}) \\ -2\Phi_{2-} \end{bmatrix} = m \begin{bmatrix} (\Phi_1 - \Phi_4)_+ \\ (\Phi_1 - \Phi_4)_0 \\ (\Phi_1 - \Phi_4)_- \end{bmatrix} , \tag{12}$$

$$\frac{k}{a} \begin{bmatrix} (\Phi_1 + \Phi_4)_+ \\ 0 \cdot (\Phi_1 + \Phi_4)_0 \\ (\Phi_1 + \Phi_4)_- \end{bmatrix} = m \begin{bmatrix} 2\Phi_{2+} \\ (\Phi_{20} + \Phi_{2\tilde{0}}) \\ 2\Phi_{2-} \end{bmatrix} \tag{13}$$

$$-\frac{k}{a}\left(\Phi_1 + \Phi_4\right)_0 = m\left(\Phi_{20} - \Phi_{2\tilde{0}}\right) . \tag{14}$$

In (13), we see that the $(\Phi_{20} + \Phi_{2\tilde{0}})$ is identically zero for the massive particles. However, it is arbitrary for the massless particles and we therefore identify this arbitrariness as the gauge freedom.

We obtain the following three equations for the longitudinal (zero) helicity states:

$$ia\left(\partial_t + 2\frac{\dot{a}}{a}\right)(\Phi_1 - \Phi_4)_0 - \left(ma + \frac{k^2}{ma}\right)(\Phi_1 + \Phi_4)_0 = 0, \tag{15}$$

$$i\left(\partial_t + \frac{\dot{a}}{a}\right)(\Phi_1 + \Phi_4)_0 - m(\Phi_1 - \Phi_4)_0 = 0, \tag{16}$$

$$(\Phi_{20} - \Phi_{2\tilde{0}}) = -\frac{k}{ma}(\Phi_1 + \Phi_4)_0 \,. \tag{17}$$

The remaining six equations are

$$i\left(\partial_t + 2\frac{\dot{a}}{a}\right)(\Phi_1 - \Phi_4)_\pm = m(\Phi_1 + \Phi_4)_\pm \,, \tag{18}$$

$$ia\left(\partial_t + \frac{\dot{a}}{a}\right)(\Phi_1 + \Phi_4)_\pm - \left(ma + \frac{k^2}{ma}\right)(\Phi_1 - \Phi_4)_\pm = 0, \tag{19}$$

$$k(\Phi_1 - \Phi_4)_\pm = ma(2\Phi_{2\pm}) \,. \tag{20}$$

These are the wave equations for the transverse, ± 1-helicity states.

3 The solutions

If we consider these equations in the expanding universe, $a(t) = a_0 \exp(Ht)$, where H is the Hubble constant and $-\infty < t < \infty$, then the equation for the zero-helicity state becomes

$$\left[\left(\partial_t + \frac{3H}{2}\right)^2 - \frac{H^2}{4} + \frac{4k^2}{a_0^2}e^{-2Ht} + 4m^2\right](\Phi_1 + \Phi_4)_0 = 0 \,. \tag{21}$$

The solution of (21) is

$$(\Phi_1 + \Phi_4)_0 = N_0 z^{\frac{3}{2}} J_{i\nu}(z) \,, \tag{22}$$

where z and ν are

$$z = \frac{2k}{a_0 H}e^{-Ht} \,,$$

$$\nu = \pm\left(\frac{4m^2}{H^2} - \frac{1}{4}\right)^{1/2} .$$

The remaining components of the zero-helicity state are derived by considering the derivatives of the Bessel functions:

$$(\Phi_1 - \Phi_4)_0 = -iH\frac{N_0}{m}z^{\frac{3}{2}}\left[\left(\frac{1}{2} - i\nu\right)J_{i\nu}(z) + zJ_{i\nu-1}(z)\right] \tag{23}$$

and

$$(\Phi_{20} - \Phi_{2\tilde{0}}) = -\frac{kN_0}{ma_0}z^{\frac{5}{2}}J_{i\nu}(z) \,. \tag{24}$$

We also obtain the same equations for the ± 1-helicity states in the background:

$$\left[\left(\partial_t + \frac{3H}{2}\right)^2 - \frac{H^2}{4} + \frac{4k^2}{a_0^2}e^{-2Ht} + 4m^2\right](\Phi_1 - \Phi_4)_\pm = 0 \,. \tag{25}$$

In a similar way, the solutions are

$$(\Phi_1 - \Phi_4)_\pm = N_\pm z^{\frac{3}{2}} J_{i\nu}(z) \,. \tag{26}$$

The remaining components of the ± 1-helicity states are derived by considering the derivatives of the Bessel functions:

$$(2\Phi_{2\pm}) = \frac{kN_\pm}{ma_0}z^{\frac{5}{2}}J_{i\nu}(z), \tag{27}$$

$$(\Phi_1 + \Phi_4)_\pm = \frac{iHN_\pm}{m}z^{\frac{3}{2}}\left[\left(\frac{1}{2} + i\nu\right)J_{i\nu}(z) - zJ_{i\nu-1}(z)\right], \tag{28}$$

and finally for $m \neq 0$

$$(\Phi_{20} + \Phi_{2\tilde{0}}) = 0 \,. \tag{29}$$

Using (21), (22), (24), (25), (26), and (27), we obtain all of the components of the general wavefunction, Ψ.

4 The current

To discuss the spin-1 particle creation, it is useful to consider the current. Therefore, using the (1) and its conjugate, we find

$$\partial_\mu \overline{\Psi}(\gamma^\mu \otimes 1 + 1 \otimes \gamma^\mu)\Psi = 0 \,, \tag{30}$$

where, to introduce the conjugate of the DKP equation, we define the conjugate of the Γ_μ^{DKP}, $\gamma^\mu \otimes 1 + 1 \otimes \gamma^\mu$, and ψ:

$$\begin{aligned}(\gamma^\mu \otimes 1 + 1 \otimes \gamma^\mu)^\dagger &= \gamma^0 \otimes \gamma^0[(\gamma^\mu \otimes 1 + 1 \otimes \gamma^\mu)^*]^T \gamma^0 \otimes \gamma^0 \\ &= \gamma^\mu \otimes 1 + 1 \otimes \gamma^\mu \,, \\ (\Gamma_\mu \otimes 1 + 1 \otimes \Gamma_\mu)^\dagger &= \gamma^0\left[(\Gamma_\mu)^*\right]^T \gamma^0 \\ &= -(\Gamma_\mu \otimes 1 + 1 \otimes \Gamma_\mu) \,, \\ \left[(\Psi)^*\right]^T \gamma^0 \otimes \gamma^0 &= \overline{\Psi} \,,\end{aligned}$$

respectively. Here $*$ denotes the complex conjugation. Then we find the conserved current from (30) as

$$j^{\mu}=\frac{1}{2}\overline{\Psi}\left[\gamma^{\mu}(x)\otimes 1+1\otimes\gamma^{\mu}(x)\right]\Psi\,. \tag{31}$$

Then we evaluate the components of the current:

$$j^{0}\simeq\frac{2H}{\pi m}\left|N\right|^{2}z^{3}\sinh\nu\pi, \tag{32}$$

$$j^{3}\simeq\frac{H\left|N\right|^{2}}{ma_{0}\pi}z^{4}\left[\sin 2z+\cosh\nu\pi\right]\,, \tag{33}$$

and the rest of the components are zero because we choose $\vec{k}=(0,0,k)$. In (33), j^3 is oscillating as a function of the time t. Thus, there are rapid oscillations in the current and these indicate the instabilities in the vacuum.

We determine the normalization constants, $N_{\pm,0}$ in the asymptotic expressions of the solutions of the DKP equations in the background, i.e. in the flat-space limit, it gives the usual $\delta\left(\vec{k}-\vec{k'}\right)$ normalization. The norm of ψ is defined as

$$\left(\psi_{\vec{k'}},\psi_{\vec{k}}\right)=\int d^{3}xa^{3}(t)\left(\psi_{\vec{k'}}\right)^{*T}\psi_{\vec{k}}\,.$$

Then we find the normalization constants

$$N_{\pm,0}=\left[\left(\frac{H}{2\pi k}\right)^{3}\left(1+\frac{H^{2}}{4m^{2}}\right)\frac{\pi\nu}{2\sinh\pi\nu}\right]^{-\frac{1}{2}}. \tag{34}$$

5 Conclusion

In this study we solved the spin-1 wave equation in the expanding universe and obtained the six sets of solutions which correspond to the positive and negative energy, and the three polarization states. We also derived the expression of the current and showed that j^3 is time-dependent and it has rapid oscillations at $t\to-\infty$, in the early period of inflation. Thus there is spin-1 particle creation, as in the spin-1/2 particle case [21]. The current, j^{μ}, is conserved, because $\left(\gamma^{\mu}\otimes 1+1\otimes\gamma^{\mu}\right)_{;\mu}=0$ in the RW metric.

We also want to discuss the relevance between the solutions of the massive DKP and the Proca equations with complex components in the same background, as we discussed the relevance between the simplified or massless version of the DKP equation and the Maxwell equations. We identify A_0 from (29) as

$$A_{0}=\frac{\left(\Phi_{20}+\Phi_{2\tilde{0}}\right)}{m}=0\,,$$

We therefore reorganize (15), (16), and (17) as

$$\begin{aligned}\partial_{t}\frac{ia\left(\Phi_{1}+\Phi_{4}\right)_{0}}{m}&=a\left(\Phi_{1}-\Phi_{4}\right)_{0}\\&=F_{0\|}\,,\end{aligned} \tag{35}$$

$$F_{0\|}=-a^{2}F^{0\|}\,,$$

$$\begin{aligned}-a^{2}\left(\Phi_{20}-\Phi_{2\tilde{0}}\right)&=\partial_{z}\frac{ia\left(\Phi_{1}+\Phi_{4}\right)_{0}}{m}\\&=F_{3\|}\,,\end{aligned} \tag{36}$$

$$F_{3\|}=a^{4}F^{3\|}\,,$$

$$a^{-3}\left[\partial_{t}\left(-a^{3}F^{0\|}\right)+\partial_{z}\left(-a^{3}F^{3\|}\right)\right]=m^{2}(-a^{-2})\times\frac{ia\left(\Phi_{1}+\Phi_{4}\right)_{0}}{m}\,, \tag{37}$$

$$a^{-3}\left[\partial_{t}\left(-a^{3}F^{0\|}\right)+\partial_{z}\left(-a^{3}F^{3\|}\right)\right]=m^{2}A^{\|}\,, \tag{38}$$

$$\begin{aligned}\frac{ia\left(\Phi_{1}+\Phi_{4}\right)_{0}}{m}&=A_{\|}\\&=-a^{-2}A^{\|}\,,\end{aligned} \tag{39}$$

where $A^{\|}$ is the longitudinal component of A^{μ} with $S_z=0$. In terms of the covariant derivatives (38) is

$$\nabla_{0}F^{0\|}+\nabla_{3}F^{3\|}=m^{2}A^{\|}\,, \tag{40}$$

where

$$\begin{aligned}A_{\|}&=\frac{ia\left(\Phi_{1}+\Phi_{4}\right)_{0}}{m}\\&=\frac{2k}{H}\frac{iN_{0}z^{\frac{1}{2}}J_{i\nu}(z)}{m}\,,\\F^{0\|}&=\left(\Phi_{1}-\Phi_{4}\right)_{0}\\&=-iH\frac{N_{0}}{m}z^{\frac{3}{2}}\left[\left(\frac{1}{2}-i\nu\right)J_{i\nu}(z)+zJ_{i\nu-1}(z)\right],\\F^{3\|}&=-\left(\Phi_{20}-\Phi_{2\tilde{0}}\right)\\&=\frac{kN_{0}}{ma_{0}}z^{\frac{5}{2}}J_{i\nu}(z)\,.\end{aligned} \tag{41}$$

We also reorganize the remaining six equations as

$$\begin{aligned}\left(\partial_{t}+2\frac{\dot{a}}{a}\right)\frac{i\left(\Phi_{1}-\Phi_{4}\right)_{\pm}}{m}&=\left(\Phi_{1}+\Phi_{4}\right)_{\pm}\\&=F^{0\pm}\,,\end{aligned} \tag{42}$$

$$\begin{aligned}\frac{\nabla_{3}}{a}\frac{i\left(\Phi_{1}-\Phi_{4}\right)_{\pm}}{m}&=-\left(2\Phi_{2\pm}\right)\\&=F^{3\pm}\,,\end{aligned} \tag{43}$$

$$\nabla_{0}F^{0\pm}+\nabla_{3}F^{3\pm}=m^{2}\frac{i\left(\Phi_{1}-\Phi_{4}\right)_{\pm}}{m}\,, \tag{44}$$

where we identify $\frac{i(\Phi_1-\Phi_4)_{\pm}}{m}$ as the $A^{\pm}$. They are the space-like orthogonal components of A^{μ} with $S_z=\pm1$. The solutions are

$$\begin{aligned}A^{\pm}]&=\frac{i\left(\Phi_{1}-\Phi_{4}\right)_{\pm}}{m}\\&=\frac{iN_{\pm}}{m}z^{\frac{3}{2}}J_{i\nu}(z)\,,\end{aligned}$$

$$F^{0\pm} = (\Phi_1 + \Phi_4)_\pm$$
$$= \frac{iHN_\pm}{m} z^{\frac{3}{2}} \left[\left(\frac{1}{2} + i\nu \right) J_{i\nu}(z) - z J_{i\nu-1}(z) \right] ,$$
$$F^{3\pm} = -(2\Phi_{2\pm})$$
$$= -\frac{kN_\pm}{ma_0} z^{\frac{5}{2}} J_{i\nu}(z) .$$

We see that these solutions are complex functions of their variables. There are no prescriptions for the normalizations of the solutions of the Proca equation. These solutions are normalized in a similar way to the DKP equation by normalizing $(F, F) + (mA, mA)$.

When $m^2 \to 0$ in (33), the normalization constants become

$$N_{\pm,0} \simeq \left(\frac{2m}{H} \right) \left[\left(\frac{H}{2\pi k} \right)^3 \frac{2 \sinh \pi\nu}{\pi\nu} \right]^{\frac{1}{2}} ,$$

In this limit, using the solutions of the Proca equations, we derive the three linearly independent potential functions as

$$-iA^{\|} = \frac{(\Phi_1 + \Phi_4)_0}{m}$$
$$= \left[\left(\frac{1}{\pi k} \right)^3 \frac{2H}{\pi} \right]^{\frac{1}{2}} z^{\frac{3}{2}} J_{1/2}(z) ,$$
$$-iA^{\pm} = \frac{(\Phi_1 - \Phi_4)_\pm}{m}$$
$$= \left[\left(\frac{1}{\pi k} \right)^3 \frac{2H}{\pi} \right]^{\frac{1}{2}} z^{\frac{3}{2}} J_{1/2}(z) ,$$

and the corresponding field strengths for these potential functions as

$$F^{0\|} = F^{0\pm} = \left[\left(\frac{H}{\pi k} \right)^3 \frac{2}{\pi} \right]^{\frac{1}{2}} z^{\frac{5}{2}} J_{-1/2}(z) ,$$

$$F^{3\|} = F^{3\pm} = i \frac{k}{Ha_0} \left[\left(\frac{H}{\pi k} \right)^3 \frac{2}{\pi} \right]^{\frac{1}{2}} z^{\frac{5}{2}} J_{1/2}(z) .$$

These solutions are normalized in a similar way to the DKP equation by only normalizing (F, F) without considering the (A, A) terms. They are purely real or imaginary.

In (13), $(\Phi_{20} + \Phi_{2\tilde{0}})$ is an arbitrary function for the massless particles and we identify this property as the gauge freedom for the massless particle.

The expression of the field strength tensor are the same as our previous results in [20], where we express these solutions in terms of the conformal time, $d\tau = dt/a(t)$.

Acknowledgements. This work has been supported by the Akdeniz University Scientific Research Projects Unit.

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